Differential Equations

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q1. Assertion (A): The differential equation of all circles in a plane must be of order 3.

Reason (R): If three points are non-collinear, then only one circle always passing through these points.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q2. Assertion (A): Order of the differential equation whose solution is

$y = c_1 e^{x+c_2} + c_3 e^{x+c_4}$, **is 4.**

Reason (R): Order of the differential equation is equal to the number of independent arbitrary constants mentioned in the general solution of the differential equation.

Answer : (d) Assertion (A) is false but Reason (R) is true

Q3. Assertion (A): The solution curves of the differential equation

$$\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \sqrt{\frac{1 - x^2 - y^2}{x^2 + y^2}}$$

are circles of radius 1/2

Reason (R): The substitution x = r cos θ, y = r sin θ makes the differential equation separable.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q4. Assertion (A): The equation of curve passing through (3, 9) which satisfies differential equation

$$\frac{dy}{dx} = x + \frac{1}{x^2} \text{ is } 6xy = 3x^3 + 29x - 6.$$

Reason (R): The solution of differential equation

$$\left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right)(e^x + e^{-x}) + 1 = 0$$

is $y = e^x + c_1$ or $y = -e^{-x} + c_2$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q5.

Assertion (A): 'x' is not an integrating factor for the differential equation $x\frac{dy}{dx} + 2y = e^x$.

Reason (R): $x\left(x\frac{dy}{dx} + 2y\right) = \frac{d}{dx}(x^2 y)$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q6.

Assertion (A): $x \sin x \frac{dy}{dx} + (x + x\cos x + \sin x)y$

$= \sin x, y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi} \Rightarrow \lim_{x \to 0} y(x) = \frac{1}{3}$.

Reason (R): The differential equation is linear with integrating factor $x(1 - \cos x)$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q7.

Assertion (A): If $\frac{dy}{dx} + xy = x^3y^3$, $x > 0, y \geq 0$ and $y(0) = 1$, then $y(1) = \sqrt{2}$.

Reason (R): The differential equation is linear in the dependent variable $\frac{1}{y^2}$.

Answer : (d) Assertion (A) is false but Reason (R) is true

Q8.

Assertion (A): The order of the differential equation given by $\frac{dy}{dx} + 4y = \sin x$ is 1.

Reason (R): Since the order of a differential equation is defined as the order of the highest derivative occurring in the differential equation, *i.e.*, for nth derivative $\frac{d^n y}{d^x n}$ if $n = 1$. then it's order = 1.

Given differential equation contains only $\frac{dy}{dx}$ derivative with variables and constants.

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q9.

Assertion (A): The degree of the differential equation given by $\frac{dy}{dx} = \frac{x^4 - y^4}{(x^2 + y^2)xy}$ is 1.

Reason (R): The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radicals and fraction.

The given differential equation has first order derivative which is free from radical and fraction with power = 1, thus it has a degree of 1.

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Assertion (A): Solution of the differential equation $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$ is $\frac{e^{2y}}{3} = \frac{e^{3x}}{3} + \frac{x^2}{2} + C$

Reason (R):

$$\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$$

$$\frac{dy}{dx} = e^{-2y}(e^{3x} + x^2)$$

separating the variables

$$e^{2y}dy = (e^{3x} + x^2)dx \qquad \text{[integrating]}$$

$$\int e^{2y}dy = \int (e^{3x} + x^2)dx$$

$$\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + C.$$

Ans. Option (D) is correct.

> ***Explanation:*** Assertion (A) is wrong. The correct solution is given in Reason (R).

Assertion (A): The solution of differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ is $\cos\left(\frac{y}{x}\right) = xc$

Reason (R): $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ we can clearly see that it is an homogeneous equation substituting

$$y = vx$$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v + \tan v$$

separating the variables and integrating we get

$$\int \frac{1}{\tan v}dv = \int \frac{1}{x}dx$$

$$\log(\sin v) = \log x + \log C$$

$$\sin(v) = xC$$

$$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = xC$$

is the solution where, C is constant.

Ans. Option (D) is correct.

> ***Explanation:*** Assertion (A) is wrong. The correct solution is given in Reason (R).

Assertion (A): The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx} + 5}$ are 2 and 1 respectively

Reason (R): The differential equation

$$\left(\frac{dx}{dy}\right)^3 + 2y^{1/2} = x$$

is of order 1 and degree 3.

Ans. Option (B) is correct.

> ***Explanation:*** Squaring both sides of the given differential equation,
>
> $$\left(\sqrt{\frac{d^2y}{dx^2}}\right)^2 = \left(\sqrt{\frac{dy}{dx} + 5}\right)^2$$
>
> $$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} + 5$$
>
> The highest order is 2 and its power is 1
> ∴ Order is 2, degree is 1
> Hence, Assertion (A) is true.
> The equation given in reason (R) is,
>
> $$\left(\frac{1}{\frac{dy}{dx}}\right)^3 + 2\sqrt{y} = x$$
>
> $$\Rightarrow \quad \frac{1 + 2\sqrt{y}\left(\frac{dy}{dx}\right)^3}{\left(\frac{dy}{dx}\right)^3} = x$$
>
> $$\Rightarrow \quad 1 + 2\sqrt{y}\left(\frac{dy}{dx}\right)^3 = x\left(\frac{dy}{dx}\right)^3$$
>
> Highest order is 1 and its power is 3
> ∴ Order is 1 and degree is 3.
> Hence, reason (R) is also true.

Assertion (A): The differential equation formed by eliminating a and b from $y = ae^x + be^{-x}$ is $\frac{d^2y}{dx^2} - y = 0$

Reason (R):

$$y = ae^x + be^{-x} \qquad \text{...(i)}$$

Differentiating w.r.t. 'x'

$$\frac{dy}{dx} = ae^x - be^{-x}$$

Differentiating again w.r.t. 'x'

$$\frac{d^2y}{dx^2} = ae^x + be^{-x} \qquad \text{...(ii)}$$

Subtracting eqn. (i) from eqn. (ii)

$$\frac{d^2y}{dx^2} - y = ae^x + be^{-x} - ae^x - be^{-x}$$

$$= 0$$

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).